2025 IEEE Wireless Communications and Networking Conference (WCNC): 2025 IEEE Wireless Communications and Networking Conference (WCNC) Workshops

Using Monotonic Neural Networks for Accurate and Efficient Passive Localization Performance Modeling

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2025 IEEE Wireless Communications and Networking Conference (WCNC) | 979-8-3503-6836-9/25/$31.00 ©2025 IEEE | DOI: 10.1109/WCNC61545.2025.10978599

***Abstract*—Integrated Sensing and Communication (ISAC) systems are at the forefront of next-generation wireless technologies, enhancing high-precision target localization. Accurate prediction of localization performance is crucial for the design and optimization of these systems. Traditionally, the Cramér-Rao Lower Bound (CRLB) has been used as a theoretical benchmark for estimating localization errors, but it often does not reflect actual errors encountered in practice. The Monte Carlo simulation method, while accurate, is computationally intensive and less adaptable to varying parameters. To bridge this gap, we introduce LocNet-Mono, a novel approach based on monotonic neural networks, designed specifically for predicting localization errors. This approach maintains a consistent, monotonic relationship between input and output features, addressing the shortcomings of traditional methods. Our numerical experiments validate the high accuracy and efficiency of LocNet-Mono, confirming its potential as a superior tool for performance prediction in ISAC systems.**

***Index Terms*—Passive localization, CRLB, Monte Carlo, monotonic neural network, performance prediction.**

I. Introduction

Integrated Sensing and Communication (ISAC) technology is widely recognized as a cornerstone for future wireless communication systems, particularly in 6G networks [1] [2] [3]. Passive sensing leverages existing communication signals to detect and monitor the environment and target objects. These signals may either originate directly from the target object or be reflected from surrounding objects or environmental structures [4] [5] [6]. ISAC systems require high localization accuracy in critical applications such as autonomous driving, intelligent manufacturing, and drone navigation [7]. In such scenarios, ISAC systems must ensure reliability and stability [2]. Consequently, accrately assessing the localization performance of ISAC systems is essential. To address this, traditional approaches rely on theoretical benchmarks, such as the Cramér-Rao Lower Bound (CRLB), or experimental methods like Monte Carlo simulations.

This work was supported in part by the National Nature Science Foundation of China (NSFC) under Grant 62201362, and Grant 62101350, and in part by the Shenzhen Science and Technology Program (Grant No. RCBS20221008093126071).

CRLB is a fundamental concept in estimation theory that provides a theoretical limit on the accuracy of parameter estimation [8]. It establishes a lower bound on the variance of any unbiased estimator under ideal conditions. Specifically, the variance of the unbiased estimator of the parameter is at least equal to the inverse of the Fisher information [9]. The CRLB is widely used in studies to evaluate the performance of estimation methods [10] [11] [12]. While the CRLB establishes a theoretical benchmark, practical challenges often prevent estimation methods from achieving this bound. For instance, the actual data distribution may deviate from the assumed model [13], or the noise in real systems may not follow a Gaussian distribution. As a result, the CRLB analysis, such as that in [10], often requires approximations and extensive calculations. These limitations can hinder the CRLB from accurately reflecting true localization errors in practical scenarios.

The Monte Carlo method involves conducting simulations multiple times to approximate system behavior by generating random samples from probability distributions [14]. A key advantage of the Monte Carlo method is its ability to handle complex system models and simulate diverse scenarios with varying environmental conditions and system configurations. For example, research in [15] [16] [17] employ Monte Carlo simulations to evaluate localization performance. However, challenges include the need to rerun simulations for parameter changes or new situations, which can be cumbersome and inefficient. Ensuring result reliability and accuracy requires extensive Monte Carlo simulations, leading to increased computational complexity and time consumption, especially with large datasets or large-scale scenarios. Therefore, it is crucial to balance the trade-offs between computational resources and the desired accuracy level in simulations.

Consequently, it is crucial to develop a model that can effectively and accurately measure localization performance. Machine learning technology has been extensively applied

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in the prediction field. Reference [18] introduced a method combining machine learning techniques with RAT-based localization measurements to predict a User Equipment's (UE) location while simultaneously evaluating the uncertainty of the estimated location. A recent study [19] proposed quantifying localization uncertainty in 5G architectures by using a straightforward uncertainty metric and leveraging machine learning to predict and update the uncertainty level. These studies have enhanced the accuracy and reliability of localization. Consequently, this paper adopts a novel monotonic neural network architecture to create LocNet-Mono for accurate and efficient passive localization performance prediction. The main contributions of this paper are outlined as follows:

- The multi-layer perceptron (MLP) neural network is a popular choice for predictive tasks, but it often overlooks the inherent monotonic relationship between inputs and outputs, e.g., the localization performance is propotional to signal power. To address this limitation, we introduce a novel model called LocNet-Mono, specifically designed to guarantee a monotonic relationship between inputs and output that tailored to our performance prediction task.
- Experimental results show that the proposed LocNet-Mono model achieves superior accuracy and efficiency in predicting localization errors.

This paper is organized as follows: section II introduces the received signal model and the passive localization technique based on the time difference localization algorithm. It also discusses preliminary approaches for estimating localization errors using the CRLB, Monte Carlo simulations, and MLP neural networks, highlighting their limitations in practical applications. Sections III explores the monotonic network model and its advantages. Section IV evaluates the performance prediction capability of the proposed neural network model in localization through numerical experiments. Finally, Section V summarizes this paper.

II. System Model

In this section, we briefly introduces the signal model and the passive localization method based on time difference of arrival (TDOA) approach. Then we also introduce to predict the localization errors via three preliminary approaches, i.e., the CRLB method, Monte Carlo simulation method, and the proposed neural network method via the vanilla MLP model.

A. Signal Model and Passive Localization

ISAC systems can utilize passive localization techniques to provide precise localization services. Passive localization techniques is widely employed in various scenarios such as vehicle localization, target tracking, and indoor localization. As shown in Fig. 1, a multi-base station localization scenario is considered, where the radiation source communicates with the primary station, while the auxiliary stations collaborate with the primary station to localize the radiation source. For simplicity, we consider two dimensional localization, which can be extended to three dimensions. The location of the radiation source is $\mathbf{p}^s = [x^s, y^s]^T$. Assuming there are N receiving stations, the locations are $\mathbf{p}_i^r = [x_i^r, y_i^r]^T, i = 1, \ldots, N$, then the signal received by the i-th receiving station is

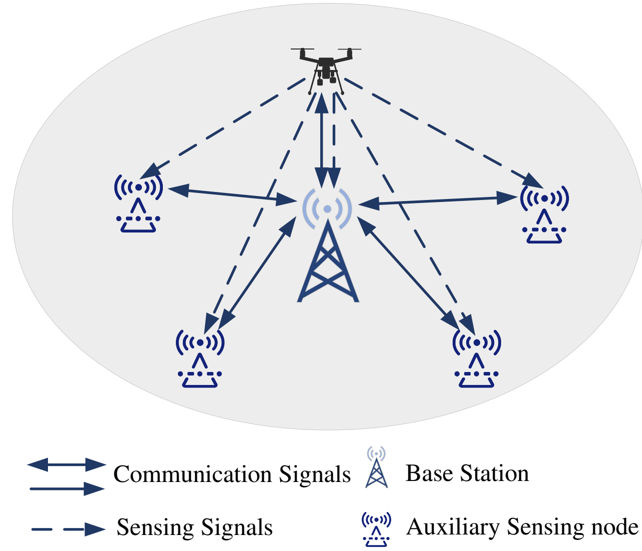

Fig. 1: Multi-base station localization scenario.

$$y_i(t) = h_i s(t - \tau_i) + n_i(t), i = 1, \ldots, N, \tag{1}$$

where h_i represents the complex channel gain from the signal source to the i-th receiving station, $s(t)$ represents the transmitted signal of the radiation source at time t, τ_i represents the time delay of the signal from the radiation source to the i-th receiving station, and $n_i(t)$ represents the noise of the i-th receiving station, which obeys a zero-mean complex Gaussian distribution.

Passive localization technology involves determining location of target without emitting electromagnetic signals, relying solely on signals emitted by the target. This typically involves a two-step process: parameter estimation and target localization. The TDOA localization algorithm calculates the location of the radiation source by measuring the time difference of signal arrival at different receiving stations. The cross-correlation function of the received signal is calculated, and its peak value is identified as the TDOA. Once the delay estimate is obtained, a system of equations is constructed and solved using the location information of each receiving station to determine the location of radiation source , as shown in (2):

$$\begin{cases} r_i = \sqrt{(x^s - x_i^r)^2 + (y^s - y_i^r)^2} \\ r_{i1} = r_i - r_1 = c \cdot \tau_{i1} \end{cases} \quad i = 1, \ldots, N, \tag{2}$$

where r_i represents the distance from the radiation source to the i-th receiving station, c is the speed of light, r_{i1} represents difference in distance between the radiation source and reference receiving station with respect to other receiving stations, and τ_{i1} represents the delay difference between the radiation source and reference receiving station with respect to other receiving stations. To solve the above equations, which are computationally complex due to their nonlinear nature, we use the Chan algorithm [20]. This algorithm employs twice-weighted least squares to predict the target location, offering a non-recursive analytical solution that eliminates the need for an initial estimate by linearizing the equations. Note that the Chan algorithm is considered to be employed by the ISAC system, it can be replaced with any other TDOA algorithm. This substitution flexibility allows us to adapt to different system requirements and preferences. In next section, we will introduce several different preliminary approaches to estimate the localization errors of the radiation source and analyze their limitations.

B. Preliminary Approaches to Predicting Localization Error

1) Experimental Approach: Monte Carlo : Monte Carlo simulations are computational algorithms that use repeated random sampling to obtain numerical results and are commonly used across various fields. This paper involves setting parameters for a specific scenario, generating extensive signal data, executing a localization algorithm to derive errors, and averaging these errors for the final result. The widely used Root Mean Square Error (RMSE) to evaluate localization errors is defined as:

$$\mathrm{RMSE}(\boldsymbol{\theta}) = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left[(x^s - \hat{x}^s)^2 + (y^s - \hat{y}^s)^2\right]}, \tag{3}$$

where $[\hat{x}^s, \hat{y}^s]^T$ is the estimated location of the radiation source. The estimated location is related to the environmental parameters used, such as power P, observation time T, baud rate f_b, and the x, y coordinates of the radiation source, and is expressed as $\boldsymbol{\theta}$. M represents the number of Monte Carlo simulations. Although straightforward, this method requires a sufficiently large M to ensure reliable and accurate results, as shown in Fig. 2. Localization errors exhibit significant fluctuations when $M = 500$. Even with $M = 2000$, fluctuations persist. A smooth curve is observed only when $M = 10,000$. This indicates that the Monte Carlo method requires a large number of simulations to produce reliable results, leading to high computational costs. The running times for $M = 500$, $M = 2000$ and$M = 10000$ on an Intel(R) Xeon(R) Gold 6252 CPU @ 2.10GHz device is 13.68min, 45.18min, and 229.76min respectively. As M increases, the running time increases. In practice, it takes a lot of time to get stable results. Furthermore, any change in parameters or conditions necessitates re-simulation, further increasing the computational burden.

2) Theoretical Approach: CRLB: CRLB is a fundamental concept in estimation theory, establishing the minimum achievable variance of an unbiased estimator for a given parameter. It is calculated as the inverse of the Fisher information matrix. The CRLB for the TDOA target estimation results, expressed as follows [8]:

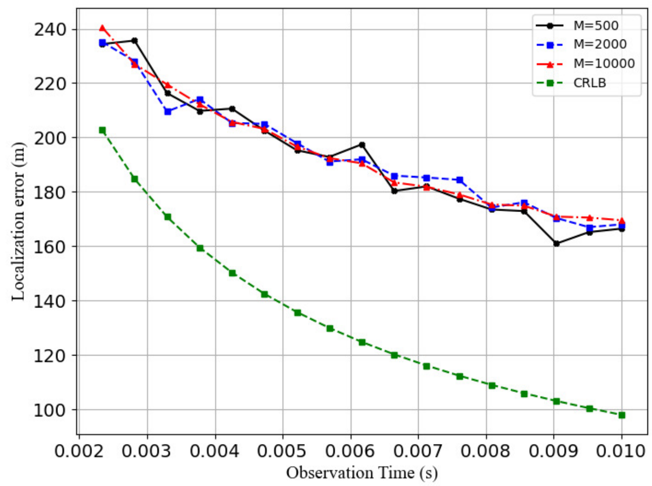

Fig. 2: An example of the RMSE obtained through Monte Carlo experiments and corresponding CRLB.

$$\mathrm{CRLB} = c^2\left(\mathbf{C}^T\mathbf{W}^{-1}\mathbf{C}\right)^{-1}, \tag{4}$$

where

$$\mathbf{C} = \begin{bmatrix} \frac{x^s - x_2^r}{r_2} - \frac{x^s - x_1^r}{r_1} & \frac{y^s - y_2^r}{r_2} - \frac{y^s - y_1^r}{r_1} \\ \frac{x^s - x_3^r}{r_3} - \frac{x^s - x_1^r}{r_1} & \frac{y^s - y_3^r}{r_3} - \frac{y^s - y_1^r}{r_1} \\ \cdots & \cdots \\ \frac{x^s - x_N^r}{r_N} - \frac{x^s - x_1^r}{r_1} & \frac{y^s - y_N^r}{r_N} - \frac{y^s - y_1^r}{r_1} \end{bmatrix}, \tag{5}$$

$\mathbf{W}$ is the covariance matrix of the localization estimation errors. The Cramér-Rao lower limit of the time difference parameter estimation is $\sigma_{TDOA} = \frac{1}{\beta\sqrt{BT\gamma}}$ [21], where β is root mean square radian frequency in the received signal spectrum, B is the noise bandwidth of the receiver, and γ is the equivalent Signal-to-Noise Ratio (SNR).

CRLB serves as a theoretical benchmark for the performance of localization algorithms. However, the gap between the CRLB and the actual errors limits the accuracy of obtaining the true localization error, as shown as Fig. 2. Additionally, the CRLB represents an ideal case, failing to account for practical system factors such as sampling rate and synchronization error. These factors complicate the accurate description of the lower bound of localization errors.

3) Neural Network: LocNet-MLP: CRLB cannot accurately measure real localization errors, and Monte Carlo methods are computationally intensive. Neural networks have gained attention for their ability to capture complex, nonlinear relationships in data, prompting many studies to apply machine learning techniques to predict the performance of various localization methods. Once trained, a machine learning model can directly predict results under new parameters or conditions without additional experiments or simulations. To predict localization performance, we apply a MLP network, which consists of an input layer, multiple hidden layers, and an output layer. The input layer receives a vector $\boldsymbol{\theta}$ of size 5, representing the effects of power, observation time, baud rate, and the x, y coordinates of the radiation source. The hidden layers are structured as $[8, 16, 8, 16, 8, 16]$, each incorporating linear transformations, batch normalization, and a ReLU activation function. These elements enhance the network's ability to learn complex patterns by introducing nonlinearities while maintaining training efficiency and stability. The output layer contains a single linear layer to produce prediction results. These fully connected layers enable the network to learn complex mappings between input features and outputs. For simplicity, we refer to this method of predicting localization error as LocNet-MLP.

Despite its effectiveness, LocNet-MLP has some limitations, particularly in ensuring output monotonicity. For example, the localization errors should decrease as the SNR increases, assuming other parameters are constant. However, the prediction results of LocNet-MLP may not be monotonic. Therefore, in the next section, we introduce the LocNet-Mono network to predict the localization errors and maintain the monotonicity of input and output.

III. Monotonic Neural Network: LocNet-Mono

Assuming all other parameters remain constant, the localization error is expected to decrease with an increase in power, observation time, and baud rate. A monotonic network ensures that the model learns the correct input-output relationship, aligning more closely with the actual system rules. Traditional MLPs lack embedded prior knowledge, such as monotonic relationships between parameters, and therefore often require large datasets to effectively learn these relationships. In contrast, the monotonic network enforces monotonicity between inputs and outputs by design, reducing the need for extensive training data and potentially improving model robustness [22] [23].

One approach involves constructing monotonic architectures, such as deep lattice networks [24], which enforce monotonicity by applying constraints within each neuron. However, this increases model complexity, potentially reducing expressiveness and affecting performance. Another approach involves heuristic and regularized architectures. Reference [25] proposed ensuring monotonicity for standard ReLU networks, but computation time significantly increases with the number of monotonic features and the model size.

A recent and novel monotonic network framework is adopted, employing a weight-constrained architecture with a single residual connection toachieve precise monotonic dependencies in any subset of the input [26]. This approach addresses the main drawbacks of limited expressiveness and impractical complexity. The architecture of the monotonic neural network is briefly described below:

For a scalar-valued function $g(\boldsymbol{\theta})$, the function satisfies Lipschitz continuity with a Lipschitz constant λ. The monotonic

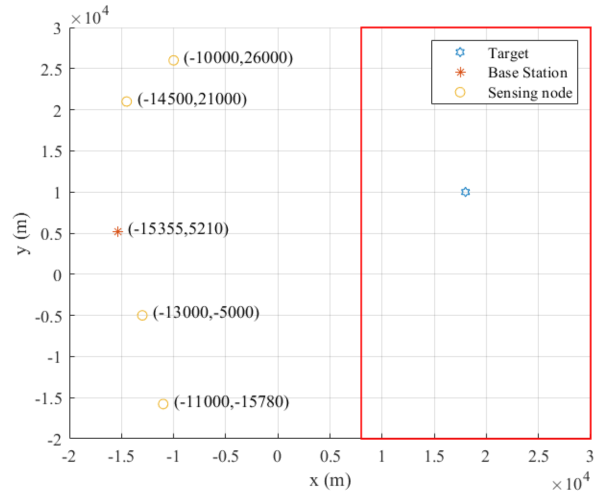

Fig. 3: Multi-base station localization scenario.

neural network architecture is constructed using the following equation:

$$f(\boldsymbol{\theta}) = g(\boldsymbol{\theta}) + \lambda \sum_{i \in \mathbb{A}} \theta_i, \tag{6}$$

where $\mathbb{A}$ denotes the set of indices of the input features for which we desire monotonicity. This residual connection enforces monotonicity. By controlling the gradient of the model $g(\boldsymbol{\theta})$, we ensure that $\frac{\partial g}{\partial \theta_i}$ lies within the interval $[-\lambda, \lambda]$ for all $i \in \mathbb{A}$. After adding the residual connection, $\frac{\partial f}{\partial \theta_i} = \frac{\partial g}{\partial \theta_i} + \lambda \geq 0$ is valid, which ensures that $f(\boldsymbol{\theta})$ is monotonically non-decreasing on the input subset $\mathbb{A}$. The direction and strength of monotonicity can be modified by changing the sign of the indicator vector. A negative indicator vector denotes a non-increasing relationship between $f(\boldsymbol{\theta})$ and $\boldsymbol{\theta}$, whereas a zero indicator vector indicates no monotonic relationship. By adjusting the parameter λ, the Lipschitz constant of the model can be increased arbitrarily.

The layer structure of the Lipschitz neural network resembles that of a MLP, with the Lipschitz constant set to $\lambda = [-\lambda, -\lambda, -\lambda, 0, 0]$. To preservethe expressiveness of the model, the GroupSort activation function [27] is employed. This function maintains a gradient norm of 1 across the entire input domain, enhancing the expressiveness of the model. We designate our monotonic neural network architecture for predicting localization performance as LocNet-Mono.

IV. Numerical Experiments

In this section, we conduct experiment to predict localization performance.

TABLE I: Data generation parameters in $\mathcal{T}$

Feature	Notation	Value
Sample Rate	f_s	10 MHz
Power	P	$[10, 70]$ W
Observation Time	T	$[0.0008, 0.01]$ s
Baud Rate	f_b	$f_s/[16,20,25,32,40,50,64,80,100]$ kHz
Target location	$\mathbf{p}^s$	$x^s \in [8, 30]$ km, $y^s \in [-20, 30]$ km

For LocNet-Mono, the hyperparameter λ is set to 10. The value of λ controls the Lipschitz constant of the neural network, thereby limiting the rate of change of each neuron's output with respect to its input. Both neural network models are trained using the Adam optimizer, with an initial learning rate of 10^{-3}. A learning rate that is too large may result in unstable training or failure to converge, whereas a learning rate that is too small can lead to excessively slow convergence. We select the ℓ_1 norm as the loss function and use the cosine annealing strategy to adjust the learning rate. This approach improves training efficiency and stability, enabling the model to converge more quickly to the optimal solution. The training epochs is set to 200. We perform 500 Monte Carlo experiments to ensure the accuracy and reliability of the statistical results.

A. Dataset Generation and Processing

The feature vectors $\boldsymbol{\theta}$ of the dataset include power, observation time, baud rate, and the x, y coordinates of the radiation source. The data labels represent the RMSE($\boldsymbol{\theta}$), which is obtained from 500 Monte Carlo experiments. For simplicity, we consider two dimensional localization, which can be extended to three dimensions. The location of the receiving station is shown in Fig. 3. The radiation source signal is modulated using Binary Phase Shift Keying (BPSK). These modulated signals are generated by the MATLAB Communications Toolbox. For each sample in dataset $\mathcal{T}$, various parameters are uniformly selected in the range as shown in Table. I.

B. Performance Prediction

In this section, we compare the performance of our proposed LocNet-MLP and LocNet-Mono in predicting localization errors.

Fig. 4 shows the predicted localization errors of LocNet-MLP and LocNet-Mono under varying power, observation time, and baud rate, while keeping other parameters unchanged, as well as the RMSE obtained from 10,000 Monte Carlo experiments and corresponding CRLB. As observed in Fig. 4, a noticeable gap exists between CRLB and the actual localization errors, with the neural network more accurately capturing the actual localization performance. Notably, LocNet-Mono demonstrates slightly better performance compared to LocNet-MLP. Additionally, as shown in Fig. 4b, LocNet-MLP occasionally exhibits non-monotonic behavior, which can result in multiple local optima during optimization and complicate the search for a global optimum. This non-monotonicity may potentially failing to satisfy the monotonicity requirements essential for practical applications. Fig. 4 show that LocNet-Mono consistently maintains the expected monotonicity, which is crucial for solving optimization problems where network output serves as the objective function. Fig. 5a presents a heat map of localization errors for a radiation source moving uniformly within the red area shown in Fig. 3, based on 10,000 Monte Carlo simulations. Fig. 5b and Fig. 5c display the prediction results of LocNet-Mono and LocNet-MLP, respectively. Compared to the Monte Carlo results, the average localization error differences for LocNet-Mono and LocNet-MLP are 5.662m and 24.768m, demonstrating that LocNet-Mono achieves better performance.

V. Conclusion

This paper concentrates on predicting passive localization performance within integrated sensing and communication systems. To overcome the limitations of traditional theoretical methods and empirical Monte Carlo simulations, we have introduced a monotonic neural network named LocNet-Mono. This model maintains a monotonic relationship between input features and outputs, effectively addressing the drawbacks of conventional approaches such as the CRLB and Monte Carlo simulations. Our numerical experiments have demonstrated that LocNet-Mono accurately and efficiently predicts localization errors across a range of parameter settings.

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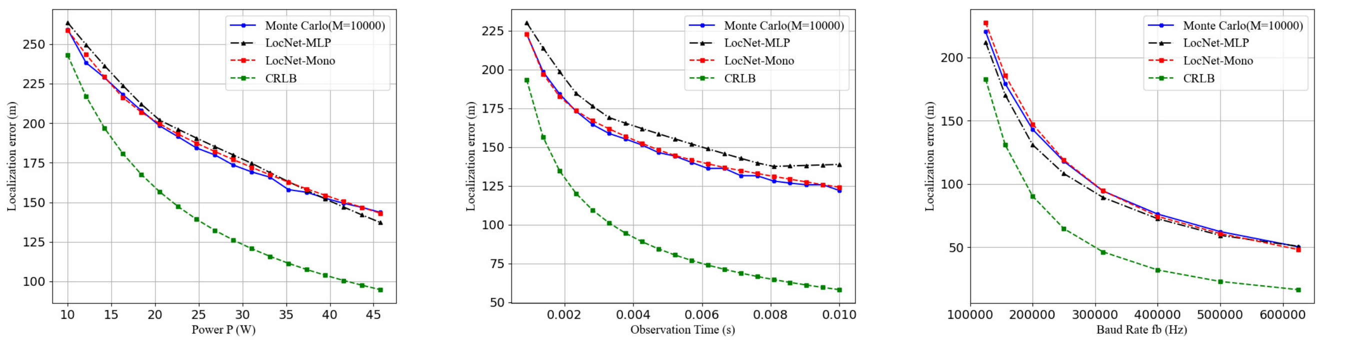
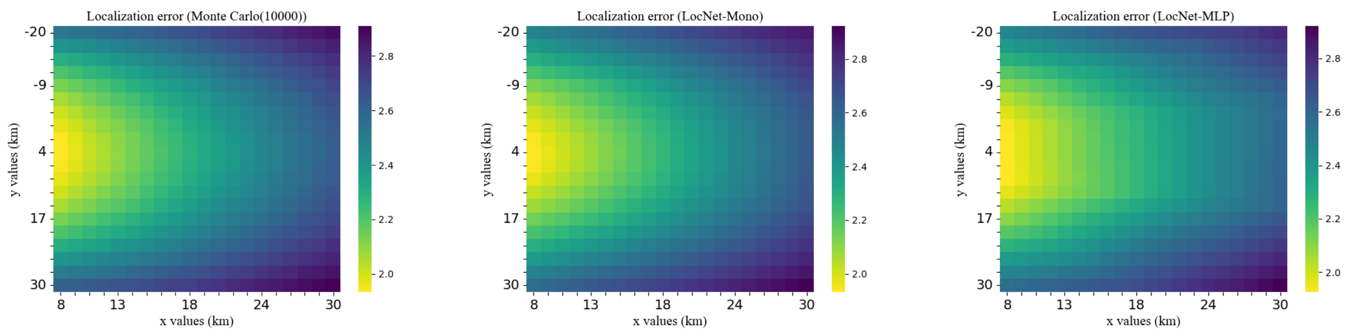


(a) Localization errors under varying P.

(b) Localization errors under varying T.

(c) Localization errors under varying f_b.

Fig. 4: The localization errors of LocNet-MLP and LocNet-Mono, with parameters: $f_s = 10$MHz, and (a) $T = 0.0009$s, $f_b = 125$kHz, $\mathbf{p}^s = [10, 3]^T$km; (b) $P = 15$W, $f_b = 125$kHz, $\mathbf{p}^s = [9, 3]^T$km; (c) $P = 40$W, $T = 0.0009$s, $\mathbf{p}^s = [9, 15]^T$km.



(a) RMSE obtained from 10,000 Monte Carlo experiments.

(b) Predicted localization errors of LocNet-Mono. Compared to Fig. 5a, the average error difference is 5.662m.

(c) Predicted localization errors of LocNet-MLP. Compared to Fig. 5a, the average error difference is 24.768m.

Fig. 5: Localization errors for the radiation source moves uniformly within the range $x \in [8, 30]$km and $y \in [-20, 30]$km, as shown in the red area of Fig. 3, with parameters: $f_s = 10$MHz, $P = 10$W, $T = 0.0009$s, $f_b = 250$kHz.

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