Efficient Position Determination Using Low-Rank Matrix Completion

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***Abstract*—In increasingly complex electromagnetic environments, distributed systems are crucial for passive target localization. These systems, consisting of spatially dispersed sensing nodes, collaboratively enhance the localization of target signals. Particularly in blind localization tasks within low signal-to-noise ratio (SNR) settings, distributed passive localization offers improved positioning performance. Traditional passive localization methods typically follow a two-step process: initially extracting parameters, such as the direction of arrivals (DOA), from raw data, followed by localizing the target. This approach often requires a high SNR. In contrast, Direct Position Determination (DPD) methods directly leverage all the received raw data, thus overcoming the constraints of the two-step process. However, DPD methods involve a time-consuming grid search within the area of interest. To address this challenge, we propose a radiation source localization method that utilizes random sampling. This method capitalizes on the low-rank properties of the grid search matrix used in DPD methods. By integrating random sampling with low-rank matrix completion algorithms, our approach efficiently localizes the radiation source. Simulation results demonstrate that this random sampling-based method significantly reduces computational demands while preserving high localization accuracy.**

***Index Terms*—Direct Position Determination (DPD), random sampling, low-rank matrix completion**

I. Introduction

In recent decades, distributed passive localization has become increasingly prevalent in both military and civilian sectors. In civilian applications, this technology is employed for radio supervision, arrest support, and search and rescue operations. In military contexts, locating radiation sources is crucial for tasks such as troop deployment, signal separation, jamming guidance, and navigational aid during attacks. Commonly used distributed passive localization methods are categorized into indirect localization algorithms and Direct Position Determination (DPD) algorithms, often described as two-step and one-step localization approaches, respectively.

Two-step localization methods are generally suboptimal because they fail to ensure that all extracted parameters are consistent with the same radiation source location. Common two-step localization techniques include received signal strength (RSS) [1], angle of arrival (AOA) [2], time difference of arrival (TDOA), frequency difference of arrival (FDOA), and systems that integrate multiple observational methods [3]. In practical scenarios, the location and velocity data of sensing nodes are often imprecise, particularly when the nodes are mounted on platforms like unmanned aerial vehicles, where random errors are inevitable. Localization based on TDOA and FDOA is particularly susceptible to inaccuracies in the position and velocity of the sensing nodes. Consequently, localization algorithms must incorporate the statistical data of these errors to improve the accuracy of radiation source localization.

In scenarios with low signal-to-noise ratio (SNR), the effectiveness of traditional two-step localization methods significantly diminishes. To overcome this limitation, DPD methods have been developed, which estimates the source location directly from the received signals, bypassing the need for intermediate parameter estimation [4]–[10]. DPD methods generally outperform the traditional two-step approaches by modeling the signal observations from all distributed sensors as functions of the same source. These methods can directly estimate the location of a radiation source from the received signals, eliminating the need for a preliminary parameter extraction step. A prevalent technique in DPD for stationary sources involves utilizing signals that encapsulate time difference and frequency difference information. DPD techniques that rely on TDOA and FDOA typically assume constant delays and Doppler shifts over the observation period. Weiss et al. conducted extensive research on DPD for narrowband radiation sources [11]. They formulated methods to estimate the radiation source location using the least squares approach, effective under both known and unknown signal conditions.

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However, the primary limitation of DPD methods is that their accuracy hinges on the grid width used to partition the search area. In cases where the search space is large and the grid width is small, the computational load can become significant, even in two-dimensional searches.

To overcome the computational challenges of grid searches, this paper introduces a novel method that combines random sampling with low-rank matrix completion (LRMC) algorithms [12] [13] [14]. This approach significantly reduces computational load while maintaining localization accuracy in complex electromagnetic environments. It enhances the efficiency of Directional Pursuit Detection (DPD) methods, proving especially effective in both military and civilian applications of distributed systems. Numerical experiments confirm that this innovative strategy not only cuts down on computational demands but also maintains high localization accuracy.

II. Signal Model and DPD

A. Signal Model

Consider a scenario where a stationary radiation source is located in a two-dimensional space, with its position denoted by $\mathbf{P}_s = (x_0, y_0)$. Surrounding this source are N stationary sensing nodes positioned at spatial coordinates $\mathbf{P}_{r,i} = (x_i, y_i)$ for $i = 1, \ldots, N$. The time delay τ_i from the radiation source to the i-th sensing node can be mathematically described as:

$$\tau_i = \frac{\|\mathbf{P}_s - \mathbf{P}_{r,i}\|_2}{c}, \quad i = 1, \ldots, N,$$

where c represents the speed of light, and $\|\cdot\|_2$ signifies the Euclidean distance between the radiation source and the i-th sensing node. To facilitate the discussion, it is assumed that the clock frequencies of the sensing nodes are synchronized. Additionally, the delays experienced during the observation period are sufficiently short, allowing them to be considered constant throughout the duration of the observation.

During the observation time slot, the received signal model used to capture the radiation source signal is defined as follows:

$$y_i(t) = h_i s(t - \tau_i) + n_i(t).$$

In this model, $y_i(t)$ represents the signal received by the i-th sensing node. The parameter h_i denotes the channel characteristic from the radiation source to the i-th node, which is assumed to follow a Rayleigh distribution. The term $s(t)$ corresponds to the radiation source signal, while τ_i indicates the delay from the radiation source to the i-th sensing node. Additionally, $n_i(t)$ is the additive white Gaussian noise at the i-th node, which follows a complex Gaussian distribution. Each node transmits observation data to the fusion center, where the received data is asynchronous. The asynchronous data received at the fusion center is denoted by $\mathbf{Y} \in \mathbb{C}^{N \times T}$, where T represents the number of observation samples per sensing node, N denotes the total number of sensing nodes.

B. DPD

The workflow of our considered Direct Position Determination (DPD) method is outlined as follows:

1) **Grid Division**: Segment the sensing area into grids.
2) **Data Compensation**: At each grid point, the fusion center adjusts the received data for time delays to derive the compensated data.
3) **Statistical Computation**: Employ various localization methods to analyze the compensated data and compute the relevant statistics.
4) **Iteration**: Repeat steps 2 and 3 for each grid point until the entire area has been covered.
5) **Source Localization**: Identify the radiation source's location at the grid point where the computed statistics reach a peak value.

Time delays can be compensated for, such that the data received by each sensing node is synchronized and free from any discrepancies in delay. The compensated data model is mathematically represented as follows:

$$\mathbf{Y} = \mathbf{H}\mathbf{s} + \mathbf{N},$$

where $\mathbf{y}(t)$ is a vector representing the signals received at time t from all N sensing nodes, and it ranges over $t = 1, \ldots, T$, where T denotes the total number of observation time points. The matrix $\mathbf{Y}$ compiles these vectors into a $N \times T$ matrix, encapsulating the data received over all time points. The matrix $\mathbf{H}$ characterizes the channel responses of each of the N nodes and remains constant across different time points. The vector $\mathbf{s}$, of dimension $1 \times T$, contains the source signals sampled across these time points. Lastly, $\mathbf{N}$ is a $N \times T$ matrix representing the noise at each node and each time point, assumed to be additive white Gaussian noise.

Our considered DPD method employs the generalized likelihood ratio test (GLRT) to address uncertainties regarding the presence of a radiation source signal. Depending on whether hypothesis $\mathcal{H}_1$ (signal presence) or $\mathcal{H}_0$ (signal absence) is assumed, the unknown parameters differ. The maximum likelihood (ML) estimations of these parameters under each hypothesis are given by:

$$\hat{\mathbf{R}}_0 = \arg\max_{\mathbf{R} \in \mathbf{\Theta}_0} p(\mathbf{Y}|\mathbf{R}, \mathcal{H}_0),$$
$$\hat{\mathbf{R}}_1 = \arg\max_{\mathbf{R} \in \mathbf{\Theta}_1} p(\mathbf{Y}|\mathbf{R}, \mathcal{H}_1),$$

where $\mathbf{\Theta}_0$ and $\mathbf{\Theta}_1$ represent the sets of unknown parameters under the hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively. Specifically, $\mathbf{\Theta}_0 := \{\mathrm{Diag}(\boldsymbol{\sigma}) | \boldsymbol{\sigma} > \mathbf{0}\}$ and $\mathbf{\Theta}_1 := \{\mathbf{H}\mathbf{H}^H + \mathrm{Diag}(\boldsymbol{\sigma}) | \boldsymbol{\sigma} > \mathbf{0}\}$ characterize the parameters under each hypothesis. Note that in this context, both $\mathbf{H}$ and $\boldsymbol{\sigma}$ are auxiliary variables and remain unknown. The generalized GLRT test statistic is formulated as:

$$\xi_{\mathrm{GLR}} = \frac{p(\mathbf{Y}|\mathbf{R}, \mathcal{H}_1)}{p(\mathbf{Y}|\mathbf{R}, \mathcal{H}_0)}.$$

For the estimation of the received signal's covariance matrix, we extend the algorithm described in [15]. This refinement allows for more accurate assessments under varying hypothesis scenarios. The DPD process ultimately determines the positions of the signal source at the grid points exhibiting the maximum test statistic, essentially pinpointing the peak of the test statistic map.

III. The Efficient DPD Based on Random Sampling.

Analyzing all grid points and computing the associated test statistics is typically time-consuming. Therefore, in this paper, we propose using the matrix completion approach to estimate the test statistics map by completing the partially observed map $\mathbf{M}$.

A. Problem Formulation

Matrix Completion (MC) is a pivotal technique in computer science that utilizes the sparsity of data—significantly zero elements in certain domains—to effectively process information. Specifically, in matrix completion, sparsity is observed in the singular value vectors of the original matrix. The main goal of MC is to reconstruct a complete, low-rank matrix from partially observed data by filling in the missing elements. The objective is to find a low-rank matrix that best matches the available partial observations, a problem which is mathematically formulated as follows:

$$\begin{aligned} \min \quad & \operatorname{rank}(\mathbf{X}) \\ \text{s.t.} \quad & \mathbf{X}_{ij} = \mathbf{M}_{ij}, \quad \forall (i,j) \in \Omega, \end{aligned} \tag{1}$$

where Ω denote the set of indices corresponding to observed elements. The constraint ensures that the constructed low-rank matrix $\mathbf{X}$ matches the known elements of matrix $\mathbf{M}$ precisely.

Minimizing the rank of a matrix is an NP-hard problem; therefore, approximation algorithms are frequently employed [12]. In the realm of sparse optimization, the l_0 norm, representing the count of non-zero singular values in a matrix, is typically substituted with the l_1 norm. Expanding on this approach, the l_1 norm is further replaced by the sum of all singular values—known as the nuclear norm of the matrix $\mathbf{X}$, denoted as $||\mathbf{X}||_* = \sum_i \sigma_i(\mathbf{X})$. Consequently, Problem (1) can be reformulated as follows:

$$\begin{aligned} \min \quad & \|\mathbf{X}\|_* \\ \text{s.t.} \quad & \mathbf{X}_{ij} = \mathbf{M}_{ij}, \quad \forall (i,j) \in \Omega, \end{aligned} \tag{2}$$

Where $||\cdot||_*$ represents the nuclear norm of a matrix, which is the sum of its singular values. Ω denotes the set of indices for observed samples, $\mathbf{X}$ represents the recovered matrix, and $\mathbf{M}$ is a matrix containing partial observations.

Problem (2) constitutes a convex optimization challenge that can be reformulated as a semidefinite programming problem. For a specified parameter $\mu > 0$, the problem adopts a quadratic penalty function form, expressed as:

$$\min_{\mathbf{X} \in \mathcal{R}^{m \times n}} \quad \mu \|\mathbf{X}\|_* + \frac{1}{2} \sum_{(i,j) \in \Omega} (\mathbf{X}_{ij} - \mathbf{M}_{ij})^2 \tag{3}$$

This formulation effectively balances the minimization of the nuclear norm and the fidelity to the observed entries of matrix $\mathbf{M}$. Moreover, alternative methods to singular value decomposition are available in matrix factorization. These approaches employ two low-rank matrices to approximate the target matrix, presupposing that the rank of the original matrix is predetermined. The objective is expressed as:

$$\min_{\mathbf{U} \in \mathcal{R}^{m \times r}, \mathbf{V} \in \mathcal{R}^{n \times r}} \|(\mathbf{U}\mathbf{V}^T)_\Omega - \mathbf{M}_\Omega\|^2$$

This formulation aims to minimize the squared Frobenius norm of the difference between the observed entries of the target matrix $\mathbf{M}_\Omega$ and the product of the approximating matrices $\mathbf{U}$ and $\mathbf{V}$.

The implementation process for the radiation source localization method utilizing random sampling is detailed as follows:

1) **Grid Division**: Segment the sensing area into grids.
2) **Sampling**: Randomly select a certain proportion of grid points.
3) **Data Compensation**: Perform time delay compensation for each sampled grid point.
4) **Statistical Computation**: Utilize various localization methods to analyze the compensated data and compute the relevant statistics.
5) **Iteration**: Repeat steps 3 and 4 for each sampled grid point until all have been processed.
6) **Matrix Completion**: Input the compiled statistical data into a matrix completion algorithm.
7) **Source Localization**: Determine the radiation source's location at the grid point corresponding to the peak value in the completed matrix.

This structured approach ensures precise localization of radiation sources within a defined area using statistical analysis and matrix algorithms.

B. Solution

To implement the Alternating Direction Method of Multipliers (ADMM) algorithm, it is necessary to modify Equation (2) by introducing an auxiliary variable $\mathbf{Z}$. The modified formulation is presented as follows:

$$\begin{aligned} \min_{\mathbf{Z}} \quad & \|\mathbf{Z}\|_* \\ \text{s.t.} \quad & P_\Omega(\mathbf{X}) = P_\Omega(\mathbf{M}), \quad \mathbf{X} = \mathbf{Z}, \end{aligned} \tag{4}$$

where $P_\Omega(\mathbf{X})_{ij} = \begin{cases} \mathbf{X}_{ij}, & \text{if } (i,j) \in \Omega, \\ 0, & \text{otherwise.} \end{cases}$ This adjustment introduces $\mathbf{Z}$ to facilitate the separation of the optimization of the nuclear norm and the constraint that $\mathbf{X}$ must match the observed entries $\mathbf{M}$ in the set Ω.

For the matrix completion problem, the objective is to estimate the unobserved elements of the true, unknown matrix $\mathbf{M}$ using its partially observed entries $P_\Omega(\mathbf{M})$. To address this, we employ a combination of the ADMM algorithm and the Singular Value Thresholding (SVT) algorithm [13]. This approach solves the optimization problem defined in (5) through an alternating iterative method.

The augmented Lagrangian function for Problem (4) is

$$\mathcal{L}(\mathbf{X}, \mathbf{Z}, \mathbf{T}) = \|\mathbf{Z}\|_* + \frac{\rho}{2}\|\mathbf{X} - \mathbf{Z}\|_F^2 + \langle \mathbf{X} - \mathbf{Z}, \mathbf{T} \rangle \quad (5)$$

where $\rho > 0$ represents a penalty factor, $\mathbf{T}$ is the Lagrange multiplier, and the notation $\langle \cdot, \cdot \rangle$ denotes the inner product. This formulation facilitates the iterative refinement of $\mathbf{X}$ and $\mathbf{Z}$, promoting convergence to the solution that best approximates the observed data.

Algorithm 1 SVT-ADMM

Input: M_Ω

Initialize: Maximum number of iterations K, $\mathbf{Z}^0$, $\mathbf{X}^0$, $\mathbf{T}^0$, ρ, and ϵ.

for $k = 1$ **to** K **do**

  STEP 1. $\mathbf{Z}^{k+1} = \mathbf{D}_{1/\rho}(\mathbf{X}^k + \frac{1}{\rho}\mathbf{T}^k)$

  STEP 2. $\mathbf{X}^{k+1} = \mathbf{P}_{1/\Omega}(\mathbf{Z}^{k+1} - \frac{1}{\rho}\mathbf{T}^k) + \mathbf{P}_\Omega(\mathbf{M})$

  STEP 3. $\mathbf{T}^{k+1} = \mathbf{T}^k - \rho(\mathbf{Z}^{k+1} - \mathbf{X}^{k+1})$

  if $\|\mathbf{X}^{k+1} - \mathbf{X}^k\|_F \leq \epsilon$ **then**

    break

  end

end

Output: $\mathbf{X}^{k+1}$

IV. Numerical Experiments

This section evaluates the matrix completion method using SVT-ADMM.We opted for a DPD method with GLRT detector. Our simulations were conducted using Matlab R2022b on a computer equipped with an AMD Ryzen 7945HX CPU. To enhance computational efficiency, a grid search was performed using parallel processing of 7 threads.

We segmented the sensing area into a grid and randomly sampled 30% of the grid points to perform low-rank matrix completion. Subsequently, we assessed and compared the time requirements and localization accuracy of various methods. The simulation scenario was established by generating the positions of radiation sources and sensing nodes within a two-dimensional space as follows:

The spatial relationship between the radiation source and the sensing nodes is illustrated in Figure 1. The actual position of the radiation source is $\mathbf{P}_s = [-5.6 \text{ km}, 8.4 \text{ km}]$, and the potential positions of the radiation sources, $\mathbf{P}_r$, are defined within the area $\{(x, y) \mid x \in [-25 \text{ km}, 25 \text{ km}], y \in [-25 \text{ km}, 25 \text{ km}]\}$. The configuration utilizes five sensing nodes, with detailed simulation parameters provided in Table I.

The vanilla DPD localization method, which processes all grid points, incurs significant computational costs and may lead to prolonged execution times. Alternatively, the random sampling-based method utilizes low-rank matrix completion techniques to reconstruct the entire matrix from a subset of observed values, thereby reducing computational demands. Utilizing the Singular Value Thresholding with Alternating Direction Method of Multipliers (SVT-ADMM) algorithm requires setting several parameters, such as the maximum number of iterations and the penalty factor. The configuration of these parameters affects the duration needed for matrix completion and, indirectly, the execution time of the localization process.

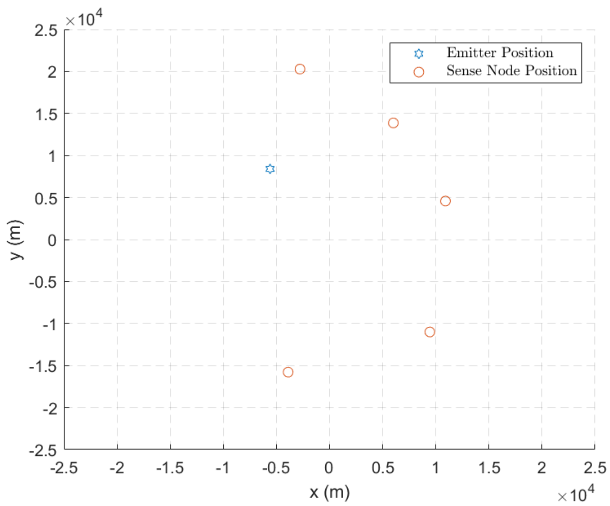


Fig. 1: Spatial distribution of emitter source and sensing nodes.

TABLE I: Simulation Setting

Parameter Name	Parameter Value
Grid Width	500m
Random Sample	30%
Modulation Type	BPSK
Baud Rate	20kHz
Reference SNR	$[-11:4:5]$dB
Sample Rate	600kHz
Observe Time	0.1s
Monte Carlo Numbers for Matrix Completion	20

The perception area is divided into grids, and the fusion center employs a localization method using GLRT, which traverses all grid points. Additionally, a localization method based on random sampling is utilized. Table II presents a comparison of the time required for different localization methods across various grid widths. It is evident that matrix completion algorithms, such as SVT-ADMM, require significantly less time than exhaustive traversal methods.

It is important to note that the comparison of time savings is approximate. In this particular simulation scenario and with the specified parameter settings, the DPD method using exhaustive traversal takes approximately five times longer than the method based on random sampling. The random sampling-based localization method necessitates careful adjustment of the parameters of the matrix completion algorithm, such as the number of iterations, to maximize its effectiveness.

TABLE II: Time consumption of DPD methods.

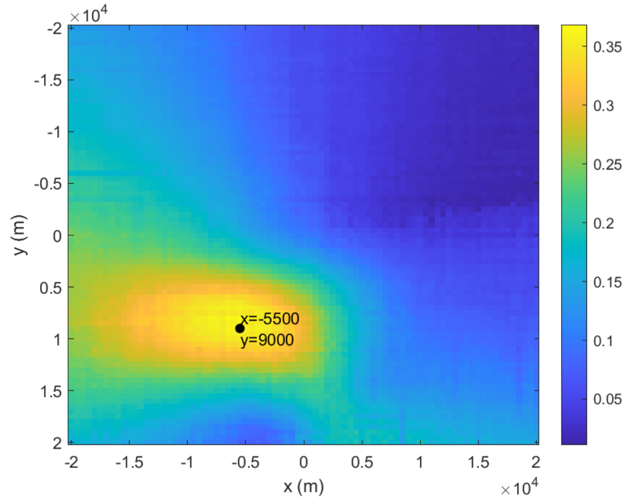
Methods \ Grid	101*101	81*81	61*61	41*41
Exhaustive grid search	26.387s	15.826s	9.383s	4.489s
Random Sample (SVT)	5.363s	3.222s	1.908s	0.911s

Figure 2 depicts the localization results using the DPD method based on GLR and the DPD method employing random sampling (SVT-ADMM), where 30% of the points are randomly sampled, and the average signal-to-noise ratio is -5.8 dB. From Figure 3, it is evident that the localization errors (Root Mean Square Error, RMSE) of the three methods are comparable when 30

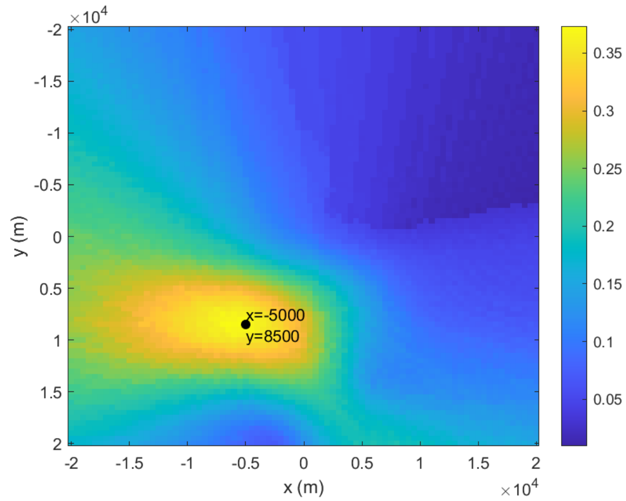
When the radiation source is both three-dimensional and mobile, the localization challenge escalates to a six-dimensional grid search, significantly increasing computational complexity. In such scenarios, the random sampling-based method demonstrates distinct advantages due to its reduced computational demands.

V. Conclusion

This work combines random sampling with low-rank matrix completion to address the computational challenges of the DPD method's grid search. This technique reduces computational demands while maintaining accuracy, as validated by numerical experiments. Future work could explore adaptive sampling schemes.



(a) GLR + SVT-ADMM



(b) GLR grid search

Fig. 2: Example of DPD results (average SNR: -5.8dB)

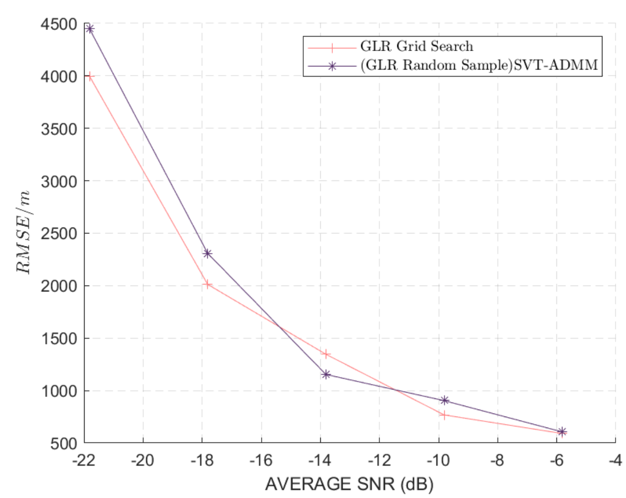


Fig. 3: Comparison of localization errors for different methods.

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