COOPERATIVE SENSING VIA MATRIX FACTORIZATION OF THE PARTIALLY RECEIVED SAMPLE COVARIANCE MATRIX

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ABSTRACT

A fundamental problem in cognitive radio is spectrum sensing, which detects the presence of the primary users in a licensed spectrum. To boost the detection performance and robustness, the multiantenna detector has been investigated and various related methods have been developed, e.g., the energy detector, the eigenvalue arithmetic-to-geometric mean detector, and the generalized likelihood ratio test detector. Cooperative sensing, which makes use of multiple receivers distributed in different locations, has the advantage of being able to make full use of the distributed antennas and enjoy a high spatial diversity gain. However, the successful employment of cooperative sensing depends on the reliable information exchange among the cooperating receivers over a long range, which may be impractical for real-world scenarios. In this paper, we consider the scenario where each receiving node can only broadcast its received raw data in a short-range communication fashion. We propose a novel cooperative sensing scheme by allowing each node to send to the fusion center only local correlation coefficients, computed within a neighborhood. A detection algorithm, based on matrix factorization of the partially received sample covariance matrix, i.e., with missing entries, is proposed. The performance of our proposed cooperative scheme is verified via numerical experiments.

***Index Terms*—** Cooperative sensing, matrix factorization, sample covariance matrix, missing entries.

1. INTRODUCTION

The cognitive radio (CR) communication and network is regarded as a promising technology for the fifth-generation (5G) wireless communication and internet of things (IoT) systems [1, 2, 3]. This technology is endowed with high spectrum efficiency and data transmission rates by means of exploiting the opportunistic spectrum access of other available networks. In a CR network, CR users are allowed to make use of the frequency band allocated to other primary users, when the latter is inactive [4]. The spectrum sensing technology is necessary for CR users to monitor the occupation status of the frequency band of interest. The cooperative sensing technique is able to make full use of the antennas that provide service to the distributed CR users and enjoy the spatial diversity gain [5, 6, 7]. For example, a group of unmanned aerial vehicles (UAVs), each of which is equipped with an omnidirectional antenna, may apply the cooperative sensing technology to detect the primary signals.

There exist several signal detection methods realizing the spectrum sensing task. Let $\mathbf{x}_t$ (a p-dimensional complex column vector) represent the received signals of p antennas at time t. The simplest energy detector uses the energy of the received signals, i.e., $\sum_t \|\mathbf{x}_t\|^2$, to decide the presence of primary signals [8]. Note that the noise power is assumed to be known apriori at the CR receivers to keep satisfactory detection performance [9]. Of course, there also exist numerous detection methods being able to detect signals without the prior knowledge of the noise variance. They are realized by taking advantage of the correlation structure in the received data. Given the eigenvalues $\{\lambda_i\}_{i=1}^p$ of the sample covariance matrix, $\mathbf{S} = \frac{1}{T}\sum_t \mathbf{x}_t\mathbf{x}_t^H$ where T is the number of samples, the eigenvalue arithmetic-to-geometric mean (AGM) detector calculates the statistic as the ratio of the eigenvalues' arithmetic mean to the geometric mean [10]. It is based on the fact that the eigenspectrum will spread out if the primary signal exists. Similar methods include the eigenvalue-moment-ratio (EMR) detector [9], the maximum-minimum eigenvalue (MME) detector [11], the scaled largest eigenvalue (SLE) detector [12], and the generalized likelihood ratio test (GLRT) detector [13]. In the cooperative sensing, all aforementioned methods are applicable if the raw data can be correctly collected from the distributed receiving nodes (CR users) to the

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fusion center.

The successful transmission, however, of raw data from the distributed receiving nodes to a shared fusion center might be impractical for real-world applications [4, 9, 14]. One reason is that in wireless communications, the transmitting power is limited due to, e.g., hardware constraints [15] or security issues [16]. In other words, the high-speed and reliable wireless data transmission is only accessible within a limited communication distance [17]. Therefore, a CR user may only share its raw data with neighboring receiving nodes, and not with the remote fusion center. In this case, all the previous signal detection methods will fail due to the absence of raw data.

In this paper, we propose a spectrum detecting scheme applicable in the aforementioned communication restricted scenario. The major contributions of this paper are as follows: *i)* we consider the cooperative spectrum sensing problem, where each receiving node can establish high-speed reliable data transmission links only with nearby nodes. To this end, we propose a cooperative sensing scheme where each receiving node broadcasts its raw data to all nearby nodes. Then, each node calculates the sample covariances between its received raw data (from nearby nodes) and that of itself, and sends them to the fusion center; *ii)* we propose a novel detection method, which is applicable when the fusion center can only receive the sample covariance matrix with missing entries. *iii)* we also propose a practical algorithm to solve the resulting optimization problem, which is to approximate the target sample covariance matrix with missing elements via the sum of a low-rank matrix plus a diagonal one.

2. PROBLEM FORMULATION

2.1. Signal Model

Consider p distributed CR users each of which is equipped with a single antenna, whose purpose is to cooperatively sense a frequency band of interest that is occupied by a primary user. The channels between the primary user and each one of the CR users are frequency non-selective (flat) fading and the rank of the respective signal subspace is assumed to be known as r. There are two hypotheses, i.e., $\mathcal{H}_0$, signal absent, and $\mathcal{H}_1$, signal present. Denote $\mathbf{x}_t \in \mathbb{C}^p$ as the vector of the received signals of these p receivers at time t. The hypothesis testing problem is written as [13]

$$\begin{aligned} \mathcal{H}_0 &: \mathbf{x}_t = \mathbf{n}_t, \\ \mathcal{H}_1 &: \mathbf{x}_t = \mathbf{H}\mathbf{s}_t + \mathbf{n}_t, \end{aligned} \tag{1}$$

where $\mathbf{s}_t \in \mathbb{C}^r$ is the primary signal following the i.i.d. zero-mean circular complex Gaussian (CCG) distribution, $\mathbf{H} \in \mathbb{C}^{p\times r}$ is the unknown channel among the primary user and the receivers, and $\mathbf{n}_t \in \mathbb{C}^p$ is the i.i.d. zero-mean CCG and uncorrelated noise.

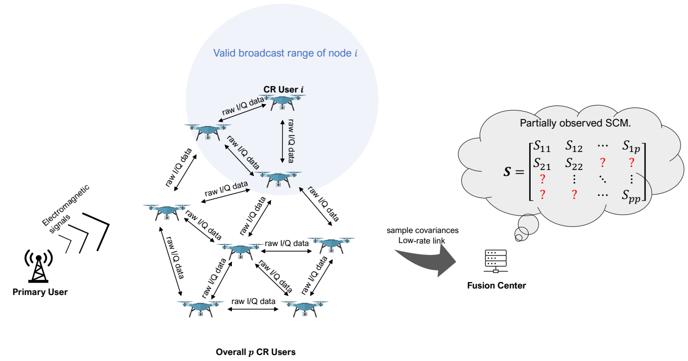

Fig. 1: The proposed distributed detection scheme under the restricted communication distance scenario.

Without loss of generality, we assume the covariance matrix of the primary signal, $\mathbf{s}$, to be identity, i.e., $\mathbb{E}\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_p$ (note that any spatial correlation and scaling of the primary signal can be absorbed in $\mathbf{H}$). Denote by $\boldsymbol{\Sigma}$ the covariance matrix of the received signal, $\mathbf{x}$. The hypothesis testing problem (1) can equivalently be written as

$$\begin{aligned} \mathcal{H}_0 &: \boldsymbol{\Sigma} = \boldsymbol{\Psi}, \\ \mathcal{H}_1 &: \boldsymbol{\Sigma} = \mathbf{H}\mathbf{H}^H + \boldsymbol{\Psi}, \end{aligned} \tag{2}$$

where $\boldsymbol{\Psi} = \operatorname{Diag}\left(\psi_1, \ldots, \psi_p\right) \succ 0$ is the covariance matrix of the noise $\mathbf{n}$. Various methods have been developed for problem (2), e.g., EMR [9], MME [11], GLR [13].

2.2. Considered Scenario

All the above methods, however, require that the raw data, i.e., $\mathbf{X} = \left[\mathbf{x}_1, \ldots, \mathbf{x}_T\right]^T$, can be correctly collected at the fusion center. As shown in Fig. 1, since the communication range of each CR user (receiving node) is limited by power constraints, the raw data in node, e.g., i can not be successful transmitted to the fusion center. Instead, the raw data are broadcasted and can be correctly received only by its geographically nearby nodes. Note that, in this case, Problem (2) becomes extremely challenging, with the difficulty stemming from the lack of raw data in the fusion center. To the best of our knowledge, this case has not been considered in the literature before and no existing methods can be employed.

In this paper, we consider a novel cooperative sensing scheme by allowing each node to transmit to the fusion center only the sample covariances between the received data from the other nodes (within its neighborhood) and that of itself. These locally computed sample covariances are transmitted to the fusion center via the low-rate but reliable communication links, e.g., via satellite communications [18]. If each node can reliably receive data from all the rest, the sample covariance matrix in the fusion center is fully formed. However, since the nodes can only access nearby nodes, the fusion center can only obtain *a partially observed sample covariance* *matrix* $\mathbf{S} \in \mathbb{C}^{p \times p}$. Let $\Omega \subset [p] \times [p]$ be a symmetric index set. We assume that $\mathbf{S}$ is only observed on the entries with index in Ω, and the diagonal elements of $\mathbf{S}$ are always collected, i.e., $(i,i) \in \Omega, i = 1, \ldots, p$. The goal of this paper is to propose a novel detector algorithm using only the partially observed matrix $\mathbf{S}$. The details are discussed in the next section.

3. FERT DETECTION ALGORITHM

In this section, we derive the matrix factorization error ratio test (FERT) detector, which solves Problem (2) under the condition of a partially observed matrix, $\mathbf{S}$. The idea is to construct the test statistic as the ratio of the matrix factorization error under hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$,

$$\xi_{\text{FER}} = \frac{f\left(\mathbf{S} \,|\, \hat{\mathbf{\Sigma}}_0, \mathcal{H}_0\right)}{f\left(\mathbf{S} \,|\, \hat{\mathbf{\Sigma}}_1, \mathcal{H}_1\right)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma, \tag{3}$$

where the threshold γ is set to provide the desired probability of false alarm, $f\left(\mathbf{S} \,|\, \mathbf{\Sigma}\right) = \|\mathcal{P}_\Omega\left(\mathbf{S} - \mathbf{\Sigma}\right)\|_F^2$ denotes the matrix approximation error, and

$$\begin{aligned} \hat{\mathbf{\Sigma}}_0 &= \arg \min_{\mathbf{\Sigma}_0 = \mathbf{\Psi}} \|\mathcal{P}_\Omega\left(\mathbf{S} - \mathbf{\Sigma}_0\right)\|_F^2 \\ \hat{\mathbf{\Sigma}}_1 &= \arg \min_{\mathbf{\Sigma}_1 = \mathbf{H}\mathbf{H}^H + \mathbf{\Psi}} \|\mathcal{P}_\Omega\left(\mathbf{S} - \mathbf{\Sigma}_1\right)\|_F^2, \end{aligned} \tag{4}$$

where $\mathcal{P}_\Omega\left(\mathbf{Z}\right)_{ij} = \begin{cases} Z_{ij} & (i,j) \in \Omega \\ 0 & (i,j) \notin \Omega \end{cases}$. The FERT decides $\mathcal{H}_1$ if $\xi_{\text{FER}} > \gamma$. The threshold γ can be set by simulating an empirical distribution, formed when the signal is known to be absent.

It is obvious that $\hat{\mathbf{\Sigma}}_0 = \text{Diag}\left(\mathbf{S}\right)$. However, $\hat{\mathbf{\Sigma}}_1$ is not easy to obtain since the corresponding problem is an non-convex one for $\mathbf{H}$. Therefore, in the following part, we propose an optimization algorithm for obtaining $\hat{\mathbf{\Sigma}}_1$ in (4).

3.1. An Alternating Optimization Algorithm

Consider problem (4) for $\hat{\mathbf{\Sigma}}_1$ and reformulate it as

$$\begin{aligned} \min_{\mathbf{H}, \mathbf{\Psi}} \quad & f(\mathbf{H}, \mathbf{\Psi}) \coloneqq \left\|\mathcal{P}_\Omega\left(\mathbf{S} - \mathbf{H}\mathbf{H}^H - \mathbf{\Psi}\right)\right\|_F^2, \\ \text{s.t.} \quad & \mathbf{H} \in \mathbb{C}^{p \times r}, \ \ \mathbf{\Psi} = \text{Diag}(\psi_1, \ldots, \psi_p) \succ \mathbf{0}. \end{aligned} \tag{5}$$

We can simply partition the variables into $\mathbf{H}$ and $\mathbf{\Psi}$, and conduct an alternating optimization scheme.

3.1.1. Update $\mathbf{H}$

Given a fixed $\mathbf{\Psi}$, the sub-problem w.r.t. $\mathbf{H}$ is written as

$$\begin{aligned} \min_{\mathbf{H} \in \mathbb{C}^{p \times r}} f(\mathbf{H}) &\coloneqq \left\|\mathcal{P}_\Omega\left(\hat{\mathbf{S}} - \mathbf{H}\mathbf{H}^H\right)\right\|_F^2 \\ &\coloneqq \sum_{(i,j) \in \Omega} \left|\mathbf{h}_i^H \mathbf{h}_j - \hat{S}_{ij}\right|^2, \end{aligned} \tag{6}$$

where $\hat{\mathbf{S}} = \mathbf{S} - \mathbf{\Psi}$ and $\mathbf{h}_i$ is the i-th row of $\mathbf{H}$. This problem is typically a low-rank matrix factorization problem with missing data [19, 20, 21]. It can be solved via the vanilla gradient descent method, whose updating rule is

$$\mathbf{H}^{k+1} = \mathbf{H}^k - \eta \nabla f(\mathbf{H}^k), \tag{7}$$

where $\eta > 0$ is the step size and the i-th row of $\nabla f(\mathbf{H}^k)$ is calculated as

$$\nabla f(\mathbf{h}_i^k) = 4 \times \sum_{j:(i,j) \in \Omega} \left(\mathbf{h}_j^H \mathbf{h}_i - S_{ij}\right) \mathbf{h}_j. \tag{8}$$

3.1.2. Update $\mathbf{\Psi}$

Given a fixed $\mathbf{H}$, the sub-problem w.r.t. $\mathbf{\Psi}$ is written as

$$\begin{aligned} \min_{\mathbf{\Psi}} \quad & \left\|\mathcal{P}_\Omega\left(\mathbf{S} - \mathbf{H}\mathbf{H}^H - \mathbf{\Psi}\right)\right\|_F^2, \\ \text{s.t.} \quad & \mathbf{\Psi} = \text{Diag}(\psi_1, \ldots, \psi_p) \succeq \mathbf{0}. \end{aligned} \tag{9}$$

The optimal solution $\mathbf{\Psi}^\star$ is simply

$$\Psi_{ii}^\star = \left[S_{ii} - (\mathbf{H}\mathbf{H}^H)_{ii}\right]_+. \tag{10}$$

The proposed alternating minimization algorithm for problem (5) can be succinctly summarized as follows: iterativly updating $\mathbf{H}$ using (7), and updating $\mathbf{\Psi}$ using (10). Note that, in practice, we may update $\mathbf{H}$, i.e., executing (7), for only a few iterations, i.e., Algorithm 1, to reduce the time consumption.

Algorithm 1 A practical algorithm for problem (5).

1: Initialize $\mathbf{H}^0$ and η.
2: **for** $k = 0, 1, 2, \ldots$ **do**
3: Update $\mathbf{H}^{k+1} = \mathbf{H}^k - \eta \nabla f(\mathbf{H}^k)$;
4: Update $\mathbf{\Psi}^{k+1}$ as in (10);
5: Terminate when converges;
6: **end for**
7: Return $\mathbf{H}$ and $\mathbf{\Psi}$.

4. NUMERICAL EXPERIMENTS

In this section, we validate the performance of our proposed cooperative detector and optimization algorithms through numerical experiments.

Consider the case where $p = 8$ distributed CR users and each of them is equipped with a single omnidirectional antenna. The task is to detect a $r = 1$ single-antenna primary signal source. The primary signal is carrying a QPSK modulated signal with the baud rate being equal to 20kHz. The sampling rate at each receiver is 100kHz. The channel between each receiver and the source is assumed to be an independent Rician fading channel with the K-factor being 4[1].

[1]The modulated signals are generated using the MATLAB Communications Toolbox [22].

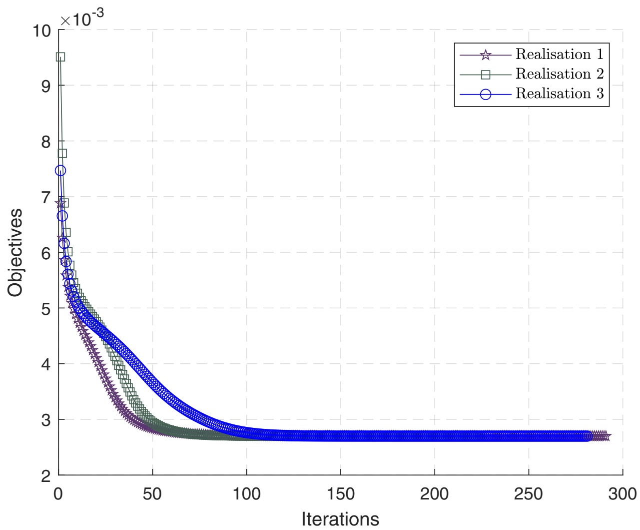

Fig. 2: An example of Algorithm 1 convergence in solving Problem (5).

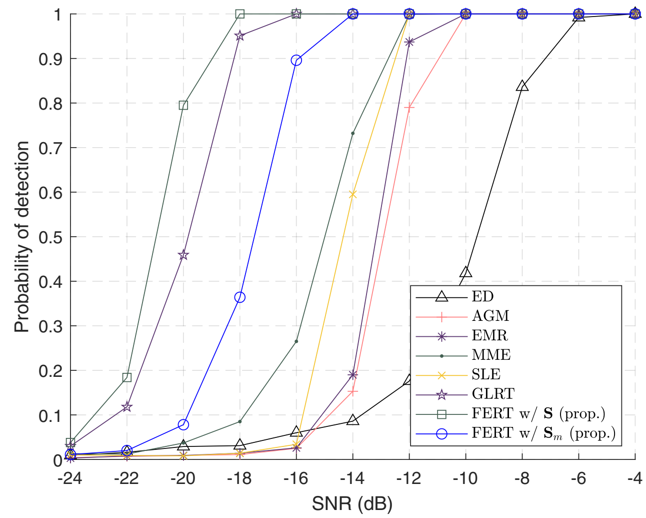

Fig. 3: Probability of detection by several detection methods.

Since the CR users are spatially distributed, we assume that the noise power is $\hat{\sigma}_i = \alpha$ ($i = 1, \ldots, p$), and α is given in dB, i.e., $10 \log_{10} \alpha$, is uniformally distributed in an interval $[-1, 1]$ [10]. The received signal-to-noise ratio (SNR) is assumed to be the same for each antenna. Each simulation experiment is conducted using $n = 10000$ continuous samples (within 100 milliseconds).

4.1. Performance of The Algorithm

We first illustrate the performance of our proposed Algorithm 1 in solving problem (5). To check the generalization of our proposed algorithm, we randomly set 50% of the elements of $\mathbf{S}$ in the fusion center to be missing. The initial point of our algorithm is $\mathbf{H}^0$ with $H^0_{ij} \sim \mathcal{CN}(0, \theta)$, where θ is the median of the off-diagonal elements of $|\mathbf{S}|$. We use the Algorithm 1 and fix the step size as $\eta = 0.5$. The algorithm terminates when the relative changes of the objective and the variables of problem (5) are less than 10^{-5}. We conduct the experiments using three random realisations of partial observations and we present the convergence of our proposed algorithm in Fig. 2. We can see that our proposed algorithm can converge well in all cases.

4.2. Detection Performance

Next, we illustrate the performance of our proposed detection method. We consider a simple communication restricted application scenario, where the i-th receiving node can only collect raw data from the j-th receiving node when $|i - j| = 1$, i.e., a partially observed sample covariance matrix $\mathbf{S}_m$ is collected at the fusion center with the observation index set $\Omega = \{(i, j) \,|\, 1 \leq i, j \leq p, |i - j| \leq 2\}$. For comparison, we also compare it with several benchmark methods and feed them with fully observed data. The test threshold, γ, is chosen to achieve probability of false alarm $P_{FA} = 1\%$. The simulation results are presented in Fig. 3. It is clear that, among all the benchmark methods, the energy detection method performs the worst and the GLRT method performs the best. This can be explained since the GLRT method is able to cope with the unequal noise variances in different receiving nodes. Our proposed FERT method can achieve similar performance as that of GLRT method if fed with the fully observed $\mathbf{S}$. The small improvement in the performance of the proposed FERT method, compared to the GLRT method, might be due to the non-convex properties of the related optimization problems. Note that all the benchmark methods require the fully received data. Therefore, it is significant that, even when fed with a partially observed sample covariance matrix, our proposed FERT method can also achieve performance better than all benchmarks except the GLRT method (which, of course, requires the full covariance matrix).

5. CONCLUSION

In this paper, we have considered the cooperative sensing problem where only local data transmission is allowed. We have proposed a matrix factorization error ratio test method to detect the presence of a signal. An alternating update based algorithm has been proposed to solve the matrix factorization problem with missing entries in the sample covariance matrix. Numerical experiments have shown the efficiency of our algorithm and the effectiveness of our cooperative sensing scheme.

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