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Efficient and Robust Cooperative Detection for Monotonically Incomplete Data

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Abstract: Cooperative spectrum sensing (CSS) in cognitive radio (CR) networks plays a crucial role in enhancing spectrum efficiency and overall network functionality. The CSS technique involves aggregating sampled data from geographically distributed CR users, which is then transmitted to a central fusion center (FC) via wireless communication. However, the diverse distances of CR users from the FC result in varying rates. Given the real-time demands of spectrum sensing tasks, it may be impractical for all CR users to complete their data transmissions in time for the final decision-making process. Consequently, the data collected at the FC is often monotonically incomplete, i.e., once a data sample from a user is missing, subsequent samples from that user are also absent. In this paper, we introduce a robust detector for CSS designed to handle monotonically incomplete data. This detector initially estimates the covariance matrix of the incomplete data and subsequently applies a classical covariance matrix-based method to detect the presence of the primary signal. We derive a closed-form solution for the corresponding maximum likelihood estimation problem, significantly enhancing the computational efficiency of the detector. Through numerical experiments, we demonstrate the computational efficiency and robustness of our proposed CSS detector.

Key Words: Cooperative spectrum sensing, monotonic incomplete data, robust detection, closed-form solution.

1 Introduction

Integrated communication and control in modern wireless networks is crucial for enhancing system efficiency and ensuring robust performance. In particular, the emergence of 6G technologies presents unprecedented opportunities for seamless integration between communication and control frameworks. Traditional control systems and wireless communication networks have been developed independently, leading to inefficiencies in real-time applications. By leveraging 6G capabilities such as ultra-reliable low-latency communication and massive connectivity, new integrated designs can significantly improve control system responsiveness and robustness [1]. These advancements enable real-time data exchange, fostering higher levels of automation and intelligent decision-making in various domains [2].

A fundamental challenge in integrated communication and control is ensuring reliable data transmission, especially in complex wireless environments. Incomplete data can significantly degrade control system performance, leading to instability or suboptimal decisions [3]. Two widely used approaches for handling missing data in such systems are deletion and imputation [4, 5]. While deletion removes incomplete observations, imputation estimates missing values to restore the dataset. However, both methods have limitations: deletion may discard critical information, whereas poor imputation can introduce bias, affecting control accuracy.

Recent research has focused on developing robust estimation techniques to handle incomplete data. For instance, robust detectors utilizing expectation-maximization (EM) algorithms have been proposed for processing missing observations [6, 7]. These methods estimate the covariance matrix of available data and apply detection algorithms accordingly. However, EM-based approaches often suffer from high computational complexity, particularly due to matrix inversion operations, which hinder their real-time applicability [8].

While missing data is often assumed to be random [9], structured missing patterns such as monotonically missing data can frequently occur in 6G-based control systems [10]. For example, in time-sensitive applications, delays in sensor-to-controller transmissions may cause sequential data loss, resulting in a monotonically missing pattern at the fusion center. Figure 1 illustrates the difference between random and monotonic missing patterns. Similar structured missing data scenarios are also observed in financial applications, where specific missing structures have been exploited to improve computational efficiency [11].

This paper introduces a CSS approach specifically engineered to handle monotonically incomplete data [12], aiming to construct a robust detector that effectively manages such challenges. The contributions of this paper are outlined as follows:

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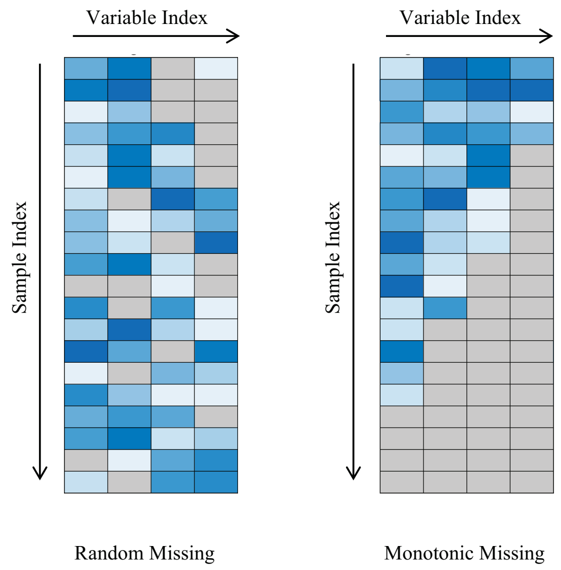

Fig. 1: An illustration of random versus monotonic missing patterns, where gray blocks indicate missing components.

- We develop a robust detector designed specifically for monotonically incomplete data. The detector first estimates the covariance matrix using the incomplete data and then employs a classical covariance matrix-based method to detect the presence of the primary signal.
- We present a closed-form solution for the associated maximum likelihood estimation problem.
- Numerical experiments are conducted to demonstrate both the computational efficiency and the robustness of our proposed detector in handling monotonically missing data.

The remainder of this paper is structured as follows. Section 2 introduces the problem formulation, focusing on integrated communication and control under structured missing data. Section 3 details our proposed estimation and detection framework. Section 4 presents numerical experiments, followed by conclusions in Section 5.

2 Problem Formulation

Consider a scenario in which a CR network system comprises $p > 1$ CR nodes. The signals transmitted by the primary user and received by the CR nodes are denoted and organized into a matrix $\mathbf{S} \in \mathbb{C}^{n \times p}$, where n represents the total number of consecutive time slots. The received signal matrix $\mathbf{X} \in \mathbb{C}^{n \times p}$ includes noise and potentially the primary user's signal $\mathbf{S}$. The spectrum sensing task is framed as a binary hypothesis testing problem, aiming to distinguish between $\mathcal{H}_0 : \mathbf{X} = \mathbf{V}$ and $\mathcal{H}_1 : \mathbf{X} = \mathbf{S}+\mathbf{V}$. The noise component $\mathbf{V}$ is generally assumed to be independent and identically distributed (i.i.d.) zero-mean circular complex Gaussian (CCG) and uncorrelated, while correlation typically exists within the primary user's signal $\mathbf{S}$. Denoting $\boldsymbol{\Sigma}$ as the covariance matrix of the received signal, then the hypothesis testing problem can be reformulated as [13]

$$\mathcal{H}_0 : \boldsymbol{\Sigma} \text{ is diagonal}, \quad \mathcal{H}_1 : \boldsymbol{\Sigma} \text{ is not diagonal}. \tag{1}$$

Typically, the existing CSS approach involves two steps: the first stage obtains the estimated covariance matrix $\hat{\boldsymbol{\Sigma}}$ from the observed data $\mathbf{X}$, and the second stage performs the CSS detection from $\hat{\boldsymbol{\Sigma}}$ utilizing classical methods such as covariance absolute value (CAV) detectors [14].

In this paper, we are interested in the scenario where the complete data $\mathbf{X}$ is unavailable, and only a partially observed, monotonically incomplete $\mathbf{X}_\mathrm{o}$ is available. In such case, obtaining the estimated $\hat{\boldsymbol{\Sigma}}$ becomes challenging. Moreover , the maximum likelihood estimation (MLE) problem for obtaining $\hat{\boldsymbol{\Sigma}}$ is formulated in [15] as follows:

$$\min_{\boldsymbol{\Sigma} \succeq \mathbf{0}} \ \ell\left(\boldsymbol{\Sigma} | \mathbf{X}_\mathrm{o}\right) = \sum_{i=1}^{n} \left(\log\det(\boldsymbol{\Sigma}_{i,\mathrm{oo}}) + \mathbf{x}_{i,\mathrm{o}}^H \boldsymbol{\Sigma}_{i,\mathrm{oo}}^{-1} \mathbf{x}_{i,\mathrm{o}} \right), \tag{2}$$

where $\ell(\boldsymbol{\Sigma}|\mathbf{X}_\mathrm{o})$ is the negative log-likelihood of $\boldsymbol{\Sigma}$ given $\mathbf{X}_\mathrm{o}$, ignoring some irrelevant constant terms. In (2), with $\mathbf{x}_{i,\mathrm{o}} \in \mathbb{C}^{p_i}$ (p_i being the number of observed entries at time index i) is a vector collecting the observed data elements in $\mathbf{x}_i$, and $\boldsymbol{\Sigma}_{i,\mathrm{oo}} \in \mathbb{S}_+^{p_i}$ is the sub-matrix of $\boldsymbol{\Sigma}$ representing the covariance matrix of $\mathbf{x}_{i,\mathrm{o}}$, where $\mathbb{S}_+^{p_i}$ is the set of all $p_i \times p_i$ symmetric positive semidefinite matrices.

The EM algorithm has been employed to solve problem (2) by converting it into a sequence of tractable surrogate problems via the expectation step and then solving these surrogate problems [16]. However, this algorithm might not be very efficient due to the potential slow convergence of the EM algorithm [17] and the significant computational cost incurred in each expectation step, which involves n executions of matrix inversion operations. In the subsequent section, we delve into the development of a robust detector designed to efficiently handle monotonically incomplete data, aiming at overcoming the computational burdens imposed by the EM algorithm and ensure effective signal detection.

3 The Efficient and Robust Detector

In this section, we first derive a closed-form solution for problem (2) and then detail the application of the obtained covariance matrix estimator for conducting detection.

3.1 A Closed-Form Solution to Problem (2)

Denote n_i as the number of observed samples from the i-th variable, i.e., the samples collected at i-th CR node. Without loss of generality, we assume that the variables in $\mathbf{X}_\mathrm{o}$ are initially sorted such that $n_1 > n_2 > \cdots > n_p$. If $\mathbf{X}_\mathrm{o}$ does not meet this criterion, we can simply sort its columns to facilitate covariance matrix estimation and subsequently restore the original order to the estimator.

We can obtain the optimal solution to problem (2) as outlined in the following Lemma.

Lemma 1. *Let $\mathbf{L}_j(\mathbf{L}_j)^H$ represents the lower triangular Cholesky decomposition of $\mathbf{R}_j = \sum_{i=1}^{n_j} \left(\mathbf{x}_{i,(j)}\right)\left(\mathbf{x}_{i,(j)}\right)^H$, where $\mathbf{x}_{i,(j)}$ comprises the first j components of $\mathbf{x}_i$, and $\mathbf{x}_i$ is a column vector transformed from the i-th row of matrix $\mathbf{X}$.Furthermore, let $\mathbf{H} \in \mathbb{C}^{p \times p}$ be an upper triangular matrix, where the first j entries of the j-th column vector, denoted by $\mathbf{h}_j$, are defined as $\mathbf{h}_j = (\mathbf{L}_j)^{-T}\left(0, \ldots, 0, n_j^{\frac{1}{2}}\right)^T$. Then the optimal solution to problem (2) is given by:*

$$\boldsymbol{\Sigma}^\star = \left(\mathbf{H}\mathbf{H}^H\right)^{-1}. \tag{3}$$

Proof. Using the standard Cholesky decomposition, the matrix $\boldsymbol{\Sigma}$ can be expressed as $\boldsymbol{\Sigma} = \mathbf{U}^H\mathbf{U}$, where $\mathbf{U} \in \mathbb{C}^{p \times p}$ is an upper triangular matrix. Taking the inverse on both sides of $\mathbf{\Sigma} = \mathbf{U}^H\mathbf{U}$, the inverse of $\mathbf{\Sigma}$ is given by:

$$\mathbf{\Sigma}^{-1} = \mathbf{H}\mathbf{H}^H, \tag{4}$$

where $\mathbf{H} = \mathbf{U}^{-1}$ is also an upper triangular matrix. Similarly, the inverse of any principal submatrix $\mathbf{\Sigma}_{(j)}$ of $\mathbf{\Sigma}$ can be decomposed as:

$$(\mathbf{\Sigma}_{(j)})^{-1} = \mathbf{H}_{(j)}(\mathbf{H}_{(j)})^H, \tag{5}$$

where $\mathbf{H}_{(j)}$ is the upper left $j \times j$ submatrix of $\mathbf{H}$. Using this decomposition, we can reparameterize the objective function in problem (2) as:

$$\ell\left(\mathbf{\Sigma}|\mathbf{X}_{\mathrm{o}}\right) = \sum_{j=1}^{p}(\mathbf{h}_j^*)^H\mathbf{R}_j\mathbf{h}_j^* - 2\sum_{j=1}^{p} n_j \log\left(h_{j,j}^*\right), \tag{6}$$

where $\mathbf{h}_j \in \mathbb{C}^j$ represents the vector of the first j components of the j-th column of $\mathbf{H}$. $\mathbf{h}_j^*$ represent the conjugate forms of $\mathbf{h}_j$ respectively and $h_{j,j}^*$ represent the conjugate forms of $h_{j,j}$. The detailed derivation of this formulation is omitted due to page limitations.

To derive the optimal values of $\mathbf{h}_j$, we first calculate the derivative of (6) with respect to $\mathbf{h}_j$ and set it to zero:

$$\frac{\partial \ell\left(\mathbf{\Sigma}|\mathbf{X}_{\mathrm{o}}\right)}{\partial \mathbf{h}_j^*} = 2\mathbf{R}_j\mathbf{h}_j^* - 2n_j\left(0, \ldots, \frac{1}{h_{j,j}^*}\right)^T = \mathbf{0}. \tag{7}$$

Solving this equation gives:

$$\mathbf{h}_j = (\mathbf{L}_j)^{-T}\left(0, \ldots, 0, n_j^{\frac{1}{2}}\right)^T, \tag{8}$$

thereby completing the proof of Lemma 1. □

3.2 The Robust CSS Detector

After obtaining $\hat{\mathbf{\Sigma}}$ from $\mathbf{X}_{\mathrm{o}}$ as outlined in Lemma 1 , we establish the detection process using classical methods. These methods generally rely on the dispersion of the eigenspectrum to detect the presence of the primary signal. Specifically, the test statistic ξ is calculated from $\hat{\mathbf{\Sigma}}$. If $\xi \geq \gamma$, the presence of the primary signal is indicated; otherwise, the signal is deemed absent. Here, γ represents a threshold set to achieve the desired probability of a false alarm.

In more detail, let $\{\lambda_i\}_{i=1}^p$ be the eigenvalues of $\hat{\mathbf{\Sigma}}$. The eigenvalue arithmetic-to-geometric mean (AGM) detector [18] computes the statistic as the ratio of the arithmetic mean to the geometric mean of the eigenvalues. Other methods, such as the maximum-minimum eigenvalue (MME) detector [19], and the covariance absolute value (CAV) detector [20], are similarly employed. The statistics are calculated as follows:

$$\xi_{\text{AGM}} = \frac{\frac{1}{p}\sum_{i=1}^{p}\lambda_i}{(\Pi_{i=1}^{p}\lambda)^{1/p}}, \tag{9}$$

$$\xi_{\text{MME}} = \frac{\max_i \lambda_i}{\min_i \lambda_i}, \tag{10}$$

$$\xi_{\text{CAV}} = \frac{\sum_{i=1}^{p}\sum_{j=1}^{p}\left|\hat{\Sigma}_{ij}\right|}{\sum_{i=1}^{p}\left|\hat{\Sigma}_{ii}\right|}. \tag{11}$$

These statistical measures are used to determine the presence of the primary signal based on the characteristics of the estimated covariance matrix.

4 Simulation Results

In this section, we conduct numerical experiments to demonstrate the correctness and efficiency of our proposed closed-form solution to problem (2), followed by an evaluation of our proposed robust CSS detector applied to monotonically incomplete data.

The simulations are set in an environment with eight distributed CR users (i.e., $p = 8$), aiming to detect a primary signal transmitting quadrature phase shift keying (QPSK) modulated signals at a baud rate of 20 kHz. The receivers operate at a sampling rate of 100 kHz. The noise power at the CR users, denoted by $\hat{\sigma}_i^2$ $(i = 1, \ldots, p)$, is uniformly distributed within an interval of [-1,1] dB. Each simulation uses $N = 500$ consecutive samples, approximately 5 milliseconds. We manually set the monotonically missing percentages from 70% to 0% for the 1st to the 8th CR node, respectively. The thresholds for each detection method are determined from the empirical test statistics of 10,000 pure noise data realizations, aiming for a false alarm probability P_{FA} of 1%. The detection probability is assessed over 1,000 realizations.

4.1 Solving Problem (2)

We first verify the correctness and demonstrate the efficiency of our closed-form solution. Fig. 2 illustrates the final result and convergence of objectives in problem (2) relative to the CPU time for both the EM algorithm (for generally randomly missing patterns) and our solution. Our solution can achieve the same final objective in closed form as the EM algorithm but requires significantly less time, even less than the time required for one iteration by the EM algorithm.

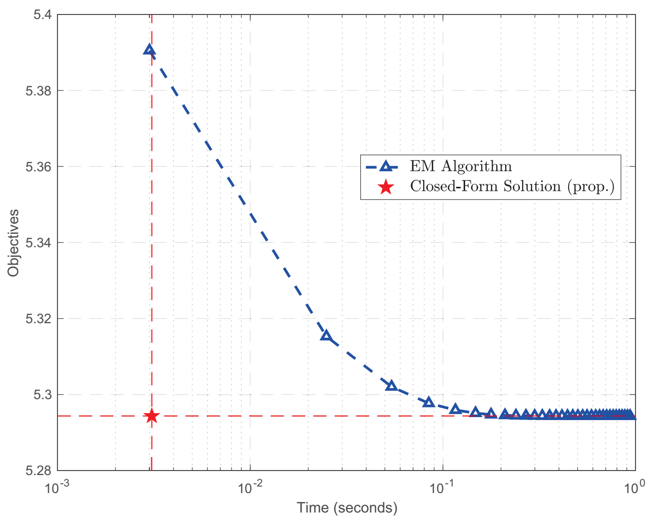

Fig. 2: Comparison of our derived closed-form solution for problem (2) with the EM algorithm applied to monotonically incomplete data.

4.2 Robustness to Monotonically Incomplete Data

We extend our investigation to analyze the detection probability P_D across a spectrum of false alarm probabilities P_{FA}. Fig. 3 presents a comparative analysis of the classical AGM, MME, and CAV detectors. The performance evaluation incorporates both the conventional sample covariance

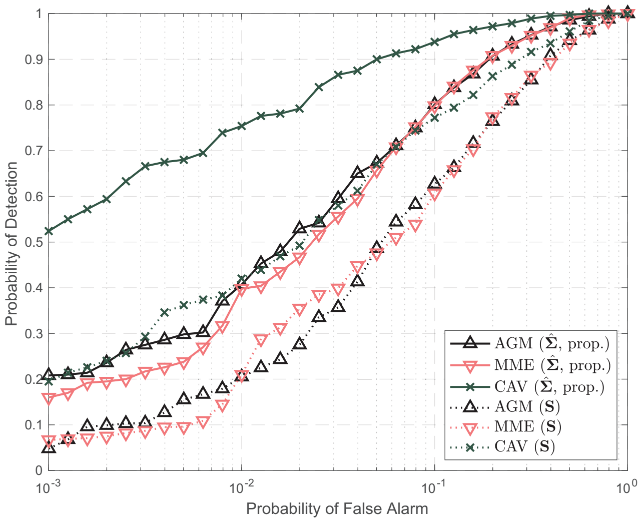

Fig. 3: P_D versus P_{FA} by several detection methods at SNR = -12 dB.

matrix $\mathbf{S}$ (after excluding samples with missing components) and the estimated $\hat{\mathbf{\Sigma}}$ obtained through our proposed closed-form solution. Remarkably, all detectors exhibit enhanced performance metrics when utilizing $\hat{\mathbf{\Sigma}}$ derived from datasets with monotonically incomplete data. This improvement underscores the superior adaptability and effectiveness of our proposed methodology in managing and compensating for data irregularities, thereby affirming its robustness and potential applicability in real-world scenarios where data incompleteness is prevalent.

5 Conclusion

In this paper, we have presented an efficient and robust cooperative spectrum sensing approach tailored for monotonically incomplete data. Our method involves initially estimating the covariance matrix using the incomplete data, followed by employing a classical covariance matrix-based technique to detect the presence of the primary signal. We have derived a closed-form solution that allows for direct covariance matrix estimation without iterations. Extensive simulations have demonstrated the computational efficiency and robustness of our proposed detection method. These results validate the efficacy of our approach and underscore its potential for practical implementation in cognitive radio systems.

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